The Batalin-Vilkovisky Field-Antifield Action for Systems with First-Class Constraints

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Abstract

The Batalin-Vilkovisky field-antifield action for systems with first-class constraints is given explicitly in terms of the canonical hamiltonian, the hamiltonian constraints and the first-order hamiltonian gauge structure functions. It is shown that this action does not depend on the hamiltonian gauge structure functions of higher orders. A method for finding the lagrangian gauge structure tensors of all orders is presented. It is proven that the lagrangian gauge structure tensors do not depend on the hamiltonian gauge structure functions of second- or higher-orders.

The Batalin-Vilkovisky (BV) quantization approach [1, 2, 3, 4] is a reliable quantization scheme for theories with open gauge algebras like supergravity [5, 6, 7]. In the lagrangian formalism, an open gauge algebra is characterized by a set of gauge structure tensors [7, 1, 2, 3]. These tensors can be obtained from the BV field-antifield action [2, 3]. In several works [8, 9, 10, 11, 12, 13, 14] it has been shown that the BV field-antifield action is determined by the BRST extended Hamiltonian and the BRST charge. In order to determine the BRST charge, one requires knowledge of the hamiltonian gauge structure functions of all orders [15]. Therefore, one may conclude that the lagrangian gauge structure tensors of higher orders must depend on the higher order hamiltonian gauge structure functions. In this paper we will show that this is not the case.

The purpose of the present paper is to prove that *all* the lagrangian gauge structure tensors are completely determined by the canonical Hamiltonian, the hamiltonian first-class constraints and the hamiltonian first-order gauge structure functions. In order to accomplish this, we will show that the generating functional of the lagrangian gauge structure tensors, the BV field-antifield action, can be written as a function of these hamiltonian quantities alone and does not depend on the hamiltonian gauge structure tensors of higher orders. We will also present an algorithm that will allow us to find all the lagrangian gauge structure tensors explicitly.

The BV field-antifield action S is a proper solution of the classical master equation [2, 3]:

$$(S, S) \equiv 0 \tag{1}$$

with the boundary conditions:

$$S\big|_{q^*=0,\mathcal{C}^*=0} = S_0 \tag{2}$$

$$\left.\frac{\delta_l \delta_r S}{\delta q_i^* \delta \mathcal{C}^\mu}\right|_{q^*=0,\mathcal{C}^*=0} = R^i_\mu \tag{3}$$

The bracket in (1) is the BV antibracket [2].

The quantities $\mathcal{C}^\mu (\mu = 1, 2, ..., m)$ are ghost fields. The variables $q_i^* (i = 1, 2, ..., n)$ and $\mathcal{C}^*_\mu (\mu = 1, 2, ..., m)$ are the antifields [2].

The zeroth-order lagrangian gauge structure function S_0 is the action functional of the physical theory:

$$S_0 = \int dt L_0 \tag{4}$$

The lagrangian first-order gauge structure functions R^i_μ are the generators of the lagrangian gauge transformations.

The BV field-antifield action can be written in the following form:

$$S = \int dt L \tag{5}$$

where [2, 3]:

$$
\begin{aligned}
L &= L_0 + q_i^* R_\alpha^i \mathcal{C}^\alpha + \frac{1}{2}\mathcal{C}_\delta^* T_{\alpha\beta}^\delta \mathcal{C}^\beta \mathcal{C}^\alpha - \frac{1}{4} q_i^* q_j^* E_{\alpha\beta}^{ji} \mathcal{C}^\beta \mathcal{C}^\alpha - \frac{1}{2}\mathcal{C}_\delta^* q_i^* D_{\alpha\beta\gamma}^{i\delta} \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha + \\
&\quad + \frac{1}{12} q_i^* q_j^* q_k^* M_{\alpha\beta\gamma}^{kji} \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha + \ldots \qquad (6)
\end{aligned}
$$

The tensors $T_{\alpha\beta}^\eta$ and $E_{\alpha\beta}^{ij}$ are the second-order gauge structure functions of the gauge algebra in the lagrangian formalism. The generators of lagrangian gauge transformations R_μ^i are said to form an open gauge algebra if $E_{\alpha\beta}^{ij} \neq 0$ [7, 1, 2, 3]. If $E_{\alpha\beta}^{ij} = 0$, the gauge algebra is said to be closed.

The existence of the higher order lagrangian gauge structure functions ($D_{\alpha\beta\gamma}^{i\rho}, M_{\alpha\beta\gamma}^{ijk}$, etc) has been proven using an axiomatic approach in [2]. These functions were constructed explicitly in [16].

The BV field-antifield action is the generating functional of the structure tensors of the gauge algebra in the lagrangian formalism [2, 3].

Let us consider a system with only primary first-class irreducible hamiltonian constraints $G_\mu (\mu = 1, 2, ..., m)$, such that [17, 18]:

$$\{G_\alpha, G_\beta\} \equiv C_{\alpha\beta}^\eta G_\eta \qquad (7)$$

$$\{H_0, G_\mu\} \equiv V_\mu^\eta G_\eta \qquad (8)$$

$$\{C_{\alpha\beta}^\eta, G_\gamma\} + \{C_{\beta\gamma}^\eta, G_\alpha\} + \{C_{\gamma\alpha}^\eta, G_\beta\} - C_{\alpha\beta}^\delta C_{\gamma\delta}^\eta - C_{\beta\gamma}^\delta C_{\alpha\delta}^\eta - C_{\gamma\alpha}^\delta C_{\beta\delta}^\eta \equiv J_{\alpha\beta\gamma}^{\eta\sigma} G_\sigma \qquad (9)$$

The function H_0 is the canonical Hamiltonian. The tensors $C_{\alpha\beta}^\eta$ and $J_{\alpha\beta\gamma}^{\eta\sigma}$ are the hamiltonian gauge structure functions of first- and second-order respectively.

Let FL^* be the pullback application [19, 20] from the momentum phase space into the velocity phase space defined by the relations:

$$\mathrm{FL}^* p_i \equiv \frac{\partial L_0}{\partial \dot{q}^i}(q, \dot{q}) \qquad (10)$$

Notice that the generators of the lagrangian gauge transformations R_μ^i can be written in terms of the hamiltonian constraints as follows [17]:

$$R_\mu^i = \mathrm{FL}^* \frac{\partial G_\mu}{\partial p_i} \qquad (11)$$

Let us consider the extended momentum phase space with points $(q^i, \mathcal{P}_i, \mathcal{C}^\alpha, \pi_\alpha, q_j^*, p^{*j}, \mathcal{C}_\beta^*, \pi^{*\beta})$, where $\mathcal{P}_i$ are the canonical momenta conjugate to q^i:

$$
\begin{aligned}
\mathcal{P}_i &= p_i + q_k^* p_{i\alpha}^k(q,p) \mathcal{C}^\alpha + \frac{1}{2}\mathcal{C}_\delta^* p_{i\alpha\beta}^\delta(q,p) \mathcal{C}^\beta \mathcal{C}^\alpha - \frac{1}{4} q_k^* q_l^* p_{i\alpha\beta}^{lk}(q,p) \mathcal{C}^\beta \mathcal{C}^\alpha \\
&\quad - \frac{1}{2}\mathcal{C}_\delta^* q_k^* p_{i\alpha\beta\gamma}^{k\delta}(q,p) \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha + \frac{1}{12} q_m^* q_l^* q_k^* p_{i\alpha\beta\gamma}^{klm}(q,p) \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha + \ldots \qquad (12)
\end{aligned}
$$

The canonical momentum variables π_α, p^{*j} and $\pi^{*\beta}$ are conjugate to the coordinates $\mathcal{C}^\alpha$, q_j^* and $\mathcal{C}_\beta^*$ respectively.

We define the pullback application $\mathcal{FL}^*$ as follows:

$$\begin{aligned}\mathcal{FL}^*\mathcal{P}_i \quad \equiv \quad & \mathrm{FL}^*p_i + q_k^*\mathrm{FL}^*p_{i\alpha}^k\mathcal{C}^\alpha + \frac{1}{2}\mathcal{C}_\delta^*\mathrm{FL}^*p_{i\alpha\beta}^\delta\mathcal{C}^\beta\mathcal{C}^\alpha - \frac{1}{4}q_k^*q_l^*\mathrm{FL}^*p_{i\alpha\beta}^{lk}\mathcal{C}^\beta\mathcal{C}^\alpha \\ & -\frac{1}{2}\mathcal{C}_\delta^*q_k^*\mathrm{FL}^*p_{i\alpha\beta\gamma}^{k\delta}\mathcal{C}^\gamma\mathcal{C}^\beta\mathcal{C}^\alpha + \frac{1}{12}q_m^*q_l^*q_k^*\mathrm{FL}^*p_{i\alpha\beta\gamma}^{klm}\mathcal{C}^\gamma\mathcal{C}^\beta\mathcal{C}^\alpha + ...\end{aligned} \tag{13}$$

$$\mathcal{FL}^*\pi_\alpha \equiv 0 \tag{14}$$

$$\mathcal{FL}^*p^{*j} \equiv 0 \tag{15}$$

$$\mathcal{FL}^*\pi^{*\beta} \equiv 0 \tag{16}$$

The functions FL^*p_i, $\mathrm{FL}^*p_{i\alpha}^k$, $\mathrm{FL}^*p_{i\alpha\beta}^\delta$, $\mathrm{FL}^*p_{i\alpha\beta}^{lk}$, $\mathrm{FL}^*p_{i\alpha\beta\gamma}^{k\delta}$, $\mathrm{FL}^*p_{i\alpha\beta\gamma}^{klm}$, etc are determined by the equations:

$$\mathcal{FL}^*\mathcal{P}_i \equiv \frac{\partial L}{\partial \dot{q}^i} \tag{17}$$

$$\mathcal{FL}^*\frac{\partial H}{\partial \mathcal{P}_i} + \Lambda^\mu\mathcal{FL}^*\frac{\partial G_\mu}{\partial \mathcal{P}_i} \equiv \dot{q}^i \tag{18}$$

$$\mathcal{FL}^*G_\mu \equiv 0 \tag{19}$$

We claim that the BV Lagrangian L can be written in terms of the canonical Hamiltonian H_0, the hamiltonian first-class constraints G_μ and the hamiltonian first-order gauge structure funtions $C_{\mu\nu}^\eta$ as follows:

$$L = \dot{q}^i\mathcal{FL}^*\mathcal{P}_i - \mathcal{FL}^*H_0 + q_k^*\mathcal{FL}^*\frac{\partial G_\alpha}{\partial \mathcal{P}_k}\mathcal{C}^\alpha + \frac{1}{2}\mathcal{C}_\delta^*\mathcal{FL}^*C_{\alpha\beta}^\delta\mathcal{C}^\beta\mathcal{C}^\alpha \tag{20}$$

The hamiltonian H will be given by the expression:

$$H = H_0\left(q,\mathcal{P}\right) - q_k^*\frac{\partial G_\alpha}{\partial \mathcal{P}_k}\left(q,\mathcal{P}\right)\mathcal{C}^\alpha - \frac{1}{2}\mathcal{C}_\delta^*C_{\alpha\beta}^\delta\left(q,\mathcal{P}\right)\mathcal{C}^\beta\mathcal{C}^\alpha \tag{21}$$

From (13) it follows that for any analytic function $K = K\left(q,\mathcal{P}\right)$ we can write:

$$
\begin{aligned}
\mathcal{FL}^* K &\equiv K\left(q, \mathcal{FL}^* \mathcal{P}\right) \\
&\equiv \mathrm{FL}^* K + q_k^* \mathrm{FL}^* \left(\frac{\partial K}{\partial p_i} p_{i\alpha}^k\right) \mathcal{C}^\alpha + \frac{1}{2} \mathcal{C}_\delta^* \mathrm{FL}^* \left(\frac{\partial K}{\partial p_i} p_{i\alpha\beta}^\delta\right) \mathcal{C}^\beta \mathcal{C}^\alpha \\
& -\frac{1}{4} q_k^* q_l^* \mathrm{FL}^* \left(\frac{\partial K}{\partial p_i} p_{i\alpha\beta}^{lk} - \frac{\partial^2 K}{\partial p_i \partial p_j} \left(p_{i\alpha}^k p_{j\beta}^l - p_{i\beta}^k p_{j\alpha}^l\right)\right) \mathcal{C}^\beta \mathcal{C}^\alpha \\
& -\frac{1}{2} \mathcal{C}_\delta^* q_k^* \mathrm{FL}^* \left(\frac{\partial K}{\partial p_i} p_{i\alpha\beta\gamma}^{k\delta} - \frac{1}{3} \frac{\partial^2 K}{\partial p_i \partial p_j} \left(p_{i\alpha}^k p_{j\beta\gamma}^\delta + p_{i\beta}^k p_{j\gamma\alpha}^\delta + p_{i\gamma}^k p_{j\alpha\beta}^\delta\right)\right) \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha \\
& +\frac{1}{12} q_n^* q_m^* q_l^* \mathrm{FL}^* \left(\frac{\partial K}{\partial p_i} p_{i\alpha\beta\gamma}^{lmn} - \frac{\partial^2 K}{\partial p_i \partial p_j} \left(p_{i\alpha}^n p_{j\beta\gamma}^{lm} + p_{i\beta}^n p_{j\gamma\alpha}^{lm} + p_{i\gamma}^n p_{j\alpha\beta}^{lm}\right)\right. \\
& \qquad \left. +2 \frac{\partial^3 K}{\partial p_i \partial p_j \partial p_k} p_{i\alpha}^n p_{j\beta}^m p_{k\gamma}^l\right) \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha + \ldots
\end{aligned} \tag{22}
$$

The Lagrange multipliers Λ^μ can also be written as:

$$
\Lambda^\mu = \lambda^\mu(q, \dot{q}) + q_k^* \lambda_\alpha^{\mu k}(q, \dot{q}) \mathcal{C}^\alpha + \frac{1}{2} \mathcal{C}_\delta^* \lambda_{\alpha\beta}^{\mu\delta}(q, \dot{q}) \mathcal{C}^\beta \mathcal{C}^\alpha + \ldots \tag{23}
$$

Let us consider the action:

$$
S = \int dt L \tag{24}
$$

where L is given by the expression (20).

We will prove that the action S (24,20) is a proper solution of the classical master equation (1) with the boundary conditions (2) and (3).

The antibracket in (1) can be written as [2, 3]:

$$
(S, S) \equiv 2 \int dt \left(\frac{\delta_r S}{\delta q^i} \frac{\delta_l S}{\delta q_i^*} + \frac{\delta_r S}{\delta \mathcal{C}^\alpha} \frac{\delta_l S}{\delta \mathcal{C}_\alpha^*}\right) \tag{25}
$$

From (20) and (21) we find that the first functional derivatives of the action S can be written in the following form:

$$
\frac{\delta_r S}{\delta q^i} = \mathcal{FL}^* \mathcal{P}_i \frac{d}{dt} - \mathcal{FL}^* \frac{\partial H}{\partial q^i} + \left(\dot{q}^j - \mathcal{FL}^* \frac{\partial H}{\partial \mathcal{P}_j}\right) \left(\frac{\partial}{\partial q^i} \left(\mathcal{FL}^* \mathcal{P}_j\right) + \frac{\partial}{\partial \dot{q}^i} \left(\mathcal{FL}^* \mathcal{P}_j\right) \frac{d}{dt}\right) \tag{26}
$$

$$
\frac{\delta_l S}{\delta q_i^*} = \mathcal{FL}^* \frac{\partial G_\alpha}{\partial \mathcal{P}_i} \mathcal{C}^\alpha + \frac{\partial_l}{\partial q_i^*} \left(\mathcal{FL}^* \mathcal{P}_j\right) \left(\dot{q}^j - \mathcal{FL}^* \frac{\partial H}{\partial \mathcal{P}_j}\right) \tag{27}
$$

$$
\frac{\delta_r S}{\delta \mathcal{C}^\alpha} = q_k^* \mathcal{FL}^* \frac{\partial G_\alpha}{\partial \mathcal{P}_k} + \mathcal{C}_\delta^* \mathcal{FL}^* C_{\alpha\beta}^\delta \mathcal{C}^\beta + \left(\dot{q}^j - \mathcal{FL}^* \frac{\partial H}{\partial \mathcal{P}_j}\right) \frac{\partial_r}{\partial \mathcal{C}^\alpha} \left(\mathcal{FL}^* \mathcal{P}_j\right) \tag{28}
$$

$$
\frac{\delta_l S}{\delta \mathcal{C}_\alpha^*} = \frac{1}{2} \mathcal{FL}^* C_{\beta\gamma}^\alpha \mathcal{C}^\gamma \mathcal{C}^\beta + \frac{\partial_l}{\partial \mathcal{C}_\alpha^*} \left(\mathcal{FL}^* \mathcal{P}_j\right) \left(\dot{q}^j - \mathcal{FL}^* \frac{\partial H}{\partial \mathcal{P}_j}\right) \tag{29}
$$

On the other hand, from (19) we find the following identities:

$$\frac{\partial}{\partial q^i}\left(\mathcal{F}\mathcal{L}^*G_\mu\right) \equiv \mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial q^i} + \frac{\partial}{\partial q^i}\left(\mathcal{F}\mathcal{L}^*\mathcal{P}_j\right)\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j} \equiv 0 \tag{30}$$

$$\frac{\partial}{\partial \dot{q}^i}\left(\mathcal{F}\mathcal{L}^*G_\mu\right) \equiv \frac{\partial}{\partial \dot{q}^i}\left(\mathcal{F}\mathcal{L}^*\mathcal{P}_j\right)\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j} \equiv 0 \tag{31}$$

$$\frac{\partial_l}{\partial q^*_i}\left(\mathcal{F}\mathcal{L}^*G_\mu\right) \equiv \frac{\partial_l}{\partial q^*_i}\left(\mathcal{F}\mathcal{L}^*\mathcal{P}_j\right)\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j} \equiv 0 \tag{32}$$

$$\frac{\partial_r}{\partial \mathcal{C}^\alpha}\left(\mathcal{F}\mathcal{L}^*G_\mu\right) \equiv \mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j}\frac{\partial_r}{\partial \mathcal{C}^\alpha}\left(\mathcal{F}\mathcal{L}^*\mathcal{P}_j\right) \equiv 0 \tag{33}$$

$$\frac{\partial_l}{\partial \mathcal{C}^*_\alpha}\left(\mathcal{F}\mathcal{L}^*G_\mu\right) \equiv \frac{\partial_l}{\partial \mathcal{C}^*_\alpha}\left(\mathcal{F}\mathcal{L}^*\mathcal{P}_j\right)\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j} \equiv 0 \tag{34}$$

Substituting (18) and (30-34) into (26-29) we finally obtain:

$$\frac{\delta_r S}{\delta q^i} = \mathcal{F}\mathcal{L}^*\mathcal{P}_i\frac{d}{dt} - \mathcal{F}\mathcal{L}^*\frac{\partial}{\partial q^i} - \Lambda^\mu\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial q^i} \tag{35}$$

$$\frac{\delta_l S}{\delta q^*_i} = \mathcal{F}\mathcal{L}^*\frac{\partial G_\alpha}{\partial \mathcal{P}_i}\mathcal{C}^\alpha \tag{36}$$

$$\frac{\delta_r S}{\delta \mathcal{C}^\alpha} = q^*_k\mathcal{F}\mathcal{L}^*\frac{\partial G_\alpha}{\partial \mathcal{P}_k} + \mathcal{C}^*_\delta\mathcal{F}\mathcal{L}^*C^\delta_{\alpha\beta}\mathcal{C}^\beta \tag{37}$$

$$\frac{\delta_l S}{\delta \mathcal{C}^*_\alpha} = \frac{1}{2}\mathcal{F}\mathcal{L}^*C^\alpha_{\beta\gamma}\mathcal{C}^\gamma\mathcal{C}^\beta \tag{38}$$

Therefore, we can write the BV antibracket of the action S with itself as follows:

$$\begin{aligned}(S,S) \;=\; & 2\int dt\left[\mathcal{F}\mathcal{L}^*\mathcal{P}_i\frac{d}{dt}\left(\mathcal{F}\mathcal{L}^*\frac{\partial G_\alpha}{\partial \mathcal{P}_i}\mathcal{C}^\alpha\right) + \frac{1}{2}q^*_j\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_j}\mathcal{F}\mathcal{L}^*C^\mu_{\alpha\beta}\mathcal{C}^\beta\mathcal{C}^\alpha\right.\\ & -\left(\mathcal{F}\mathcal{L}^*\frac{\partial H}{\partial q^i} + \Lambda^\eta\mathcal{F}\mathcal{L}^*\frac{\partial G_\eta}{\partial q^i}\right)\mathcal{F}\mathcal{L}^*\frac{\partial G_\mu}{\partial \mathcal{P}_i}\mathcal{C}^\mu\\ & \left.-\frac{1}{6}\mathcal{C}^*_\delta\mathcal{F}\mathcal{L}^*\left(C^\eta_{\alpha\beta}C^\delta_{\gamma\eta} + C^\eta_{\beta\gamma}C^\delta_{\alpha\eta} + C^\eta_{\gamma\alpha}C^\delta_{\beta\eta}\right)\mathcal{C}^\gamma\mathcal{C}^\beta\mathcal{C}^\alpha\right]\end{aligned} \tag{39}$$

From (39), integrating by parts and using the relations (18, 21) we find that the BV antibracket of the functional S with itself can be written as:

$$
\begin{aligned}
(S,S) \;\; = \;\; & 2\int dt \Bigg[-\frac{d}{dt}\left(\mathcal{FL}^* G_\alpha\right)\mathcal{C}^\alpha + \mathcal{FL}^*\{G_\alpha, H_0\}\mathcal{C}^\alpha + \Lambda^\mu \mathcal{FL}^*\{G_\alpha, G_\mu\}\mathcal{C}^\alpha \\
& -\frac{1}{2}q_k^* \mathcal{FL}^* \left(\frac{\partial}{\partial \mathcal{P}_k}\{G_\alpha, G_\beta\} - \frac{\partial G_\eta}{\partial \mathcal{P}_k} C^\eta_{\alpha\beta} \right) \mathcal{C}^\beta \mathcal{C}^\alpha \\
& -\frac{1}{6}\mathcal{C}^*_\delta \mathcal{FL}^* \left(\{G_\alpha, C^\delta_{\beta\gamma}\} + \{G_\beta, C^\delta_{\gamma\alpha}\} + \{G_\gamma, C^\delta_{\alpha\beta}\} + C^\eta_{\alpha\beta}C^\delta_{\gamma\eta} + C^\eta_{\beta\gamma}C^\delta_{\alpha\eta} + C^\eta_{\gamma\alpha}C^\delta_{\beta\eta} \right) \mathcal{C}^\gamma \mathcal{C}^\beta \mathcal{C}^\alpha \Bigg] \\
& + 2\mathcal{FL}^* \left(\mathcal{P}_i \frac{\partial G_\alpha}{\partial \mathcal{P}_i} \right) \mathcal{C}^\alpha \Bigg|_{t_i}^{t_f} \qquad (40)
\end{aligned}
$$

Using (7, 8, 9) and (19) in (40), it immediately follows that (40) reduces to:

$$
(S,S) = 2\mathcal{FL}^* \left(\mathcal{P}_i \frac{\partial G_\alpha}{\partial \mathcal{P}_i} \right) \mathcal{C}^\alpha \Bigg|_{t_i}^{t_f} \qquad (41)
$$

Assuming that $\mathcal{C}^\alpha(t_i) = \mathcal{C}^\alpha(t_f) = 0$ we finally obtain:

$$
(S,S) = 0 \qquad (42)
$$

This proves that S in (24, 20) satisfies the BV classical master equation.

From (20) and (12) it immediately follows that:

$$
L\big|_{q^*=0,\mathcal{C}^*=0} = \dot{q}^i \mathrm{FL}^* p_i - \mathrm{FL}^* H_0 = L_0 \qquad (43)
$$

and therefore, S in (24, 20) satisfies the boundary condition (2).

From (36) and (37) it follows that:

$$
\left.\frac{\delta_l \delta_r S}{\delta q_i^* \delta \mathcal{C}^\mu}\right|_{q^*=0,\mathcal{C}^*=0} = \mathcal{FL}^* \frac{\partial G_\alpha}{\partial \mathcal{P}_k} \qquad (44)
$$

Finally, from (44), (22) and (11) we obtain:

$$
\left.\frac{\delta_l \delta_r S}{\delta q_i^* \delta \mathcal{C}^\mu}\right|_{q^*=0,\mathcal{C}^*=0} = R^i_\mu \qquad (45)
$$

This proves that the action functional S also satisfies the boundary conditions (3).

Therefore, we conclude that the Batalin-Vilkovisky field-antifield action can be written in the form:

$$
S = \int dt \left[\dot{q}^i \mathcal{FL}^* \mathcal{P}_i - \mathcal{FL}^* H_0 + q_k^* \mathcal{FL}^* \frac{\partial G_\alpha}{\partial \mathcal{P}_k} \mathcal{C}^\alpha + \frac{1}{2}\mathcal{C}^*_\delta \mathcal{FL}^* C^\delta_{\alpha\beta} \mathcal{C}^\beta \mathcal{C}^\alpha \right] \qquad (46)
$$

As it can be seen from (46), the BV field-antifield action is completely determined by the canonical hamiltonian H_0, the hamiltonian constraints G_μ and the hamiltonian first-order gauge structure functions $C^\eta_{\mu\nu}$. It does not depend on the hamiltonian second-order gauge structure functions $J^{\eta\sigma}_{\alpha\beta\gamma}$ or other higher order hamiltonian gauge structure functions.

Since the BV field-antifield action is the generating functional of all the lagrangian gauge structure tensors (6), we conclude that the lagrangian gauge structure tensors of all orders are completely determined by H_0, G_μ and $C^\eta_{\mu\nu}$.

Notice that formula (46) is valid for the generic case of systems with *open* lagrangian gauge algebras. These systems may have nonvanishing lagrangian gauge structure tensors of higher orders.

From (46) using (13) and (22) and expanding (18) and (19) with the use of (13), (22) and (21, 23) we obtain the lagrangian gauge structure tensors up to fourth order in the form:

$$S_0 = \int dt \left[\dot{q}^i \mathrm{FL}^* p_i - \mathrm{FL}^* H_0\right] \tag{47a}$$

$$R^i_\mu = \mathrm{FL}^* \frac{\partial G_\mu}{\partial p_i} \tag{47b}$$

$$T^\eta_{\mu\nu} = \mathrm{FL}^* C^\eta_{\mu\nu} \tag{47c}$$

$$E^{ij}_{\mu\nu} = \mathrm{FL}^* \left(p^i_{k\mu} \frac{\partial^2 G_\nu}{\partial p_k \partial p_j} - p^i_{k\nu} \frac{\partial^2 G_\mu}{\partial p_k \partial p_j} \right) \tag{47d}$$

$$D^{i\rho}_{\alpha\beta\gamma} = -\frac{1}{3} \mathrm{FL}^* \left(p^i_{k\alpha} \frac{\partial C^\rho_{\beta\gamma}}{\partial p_k} + p^i_{k\beta} \frac{\partial C^\rho_{\gamma\alpha}}{\partial p_k} + p^i_{k\gamma} \frac{\partial C^\rho_{\alpha\beta}}{\partial p_k} \right) \tag{47e}$$

$$\begin{aligned} M^{ijk}_{\alpha\beta\gamma} = & -\frac{1}{3} \mathrm{FL}^* \left[\frac{\partial^2 G_\alpha}{\partial p_k \partial p_l} p^{ij}_{l\beta\gamma} + \frac{\partial^2 G_\beta}{\partial p_k \partial p_l} p^{ij}_{l\gamma\alpha} + \frac{\partial^2 G_\gamma}{\partial p_k \partial p_l} p^{ij}_{l\alpha\beta} + \right. \\ & + \frac{\partial^3 G_\alpha}{\partial p_k \partial p_m \partial p_n} \left(p^i_{m\beta} p^j_{n\gamma} - p^i_{m\gamma} p^j_{n\beta} \right) + \frac{\partial^3 G_\beta}{\partial p_k \partial p_m \partial p_n} \left(p^i_{m\gamma} p^j_{n\alpha} - p^i_{m\alpha} p^j_{n\gamma} \right) + \\ & \left. + \frac{\partial^3 G_\gamma}{\partial p_k \partial p_m \partial p_n} \left(p^i_{m\alpha} p^j_{n\beta} - p^i_{m\beta} p^j_{n\alpha} \right) \right] \end{aligned} \tag{47f}$$

The functions $\mathrm{FL}^* p^i_{j\alpha}$ and $\mathrm{FL}^* p^{ij}_{k\alpha\beta}$ are given by the expressions:

$$\mathrm{FL}^* p^i_{j\mu} = W_{jk} \mathrm{FL}^* \frac{\partial^2 G_\mu}{\partial p_k \partial p_i} \tag{48}$$

$$\begin{aligned} \mathrm{FL}^* p^{ij}_{k\mu\nu} = & \frac{\partial W_{lm}}{\partial \dot{q}^k} \mathrm{FL}^* \left(\frac{\partial^2 G_\mu}{\partial p_i \partial p_l} \frac{\partial^2 G_\nu}{\partial p_m \partial p_j} - \frac{\partial^2 G_\nu}{\partial p_i \partial p_l} \frac{\partial^2 G_\mu}{\partial p_m \partial p_j} \right) + \\ & + W_{lm} W_{kn} \mathrm{FL}^* \frac{\partial}{\partial p_n} \left(\frac{\partial^2 G_\mu}{\partial p_i \partial p_l} \frac{\partial^2 G_\nu}{\partial p_m \partial p_j} - \frac{\partial^2 G_\nu}{\partial p_i \partial p_l} \frac{\partial^2 G_\mu}{\partial p_m \partial p_j} \right) \end{aligned} \tag{49}$$

The above derivations illustrate how the lagrangian gauge structure tensors can be derived from the BV field-antifield action (46).

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